

STEM INSTITUTE/CCNY

PRECALCULUS

Final Examination

SUMMER 2010

Instructions: Solve any 10 complete problems. Omit 1 problem. Show all work. Calculators are not permitted.

1.

a. Given $f(x) = 1 - 2x$ and $g(x) = 2x - x^2$. Find and simplify: $f(g(x)) - g(-2)$

b. Find the exact value of: i.) $\tan\left(\cos^{-1}\left(\frac{2}{5}\right)\right)$, ii.) $\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$.

2.

a. Find the center C and the radius r of the circle with equation $x^2 + y^2 = 12x - 8y - 27$.

b. Find the x- and y- coordinates of the vertex of the parabola with equation $y = -3x^2 - 6x + 8$

3.

a. Find an equation of the line through $(-3, 5)$ that is perpendicular to the line $3x + 4y = 7$

b. Solve the inequality $|5 - 3x| \leq 2$. Write the solution in interval notation.

4.

a. Given the function $g(x) = \frac{3-2x}{5-x}$, find the inverse function $g^{-1}(x)$.

b. Solve for x : $e^{2x} - 8e^x + 15 = 0$.

5.

a. A rectangle has an area of $18 m^2$. Find a function that models its perimeter P in terms of its length x of one of its sides.

b. Solve for x : $3^{4x-1} = 5$.

6. Sketch the graph of $f(x) = -2\sin(3x + \pi)$. What are the amplitude, period, and phase shift? Label all maxima and minima with their coordinates.

7. A culture contains 80 bacteria initially. After 5 hours the bacteria count is 320. Find the exact number of bacteria after 15 hours.

8.

a. Prove the identity: $\frac{(\sin x + \cos x)^2}{\sin x \cos x} = 2 + \sec x \csc x$

b. Suppose $\sec x = -\frac{5}{3}$ and $\frac{\pi}{2} < x < \pi$. Find $\sin 2x$.

9.

a. Find the solutions of the equation $2\sin 3x - 1 = 0$ in the interval $[0, 2\pi)$.

b. Find $\log_3\left(\frac{\sqrt[4]{27}}{81}\right)$.

10.

a. Solve for x : $\log_7(x+5) = 2 + \log_7(x-3)$.

b. Solve the inequality $\frac{x}{3} < \frac{x}{x-2}$. Give your answer interval form.

11. The half-life of a radioactive substance is 48 years. If initially there are 200 grams present,

a. How much is exactly left after 96 years?

b. When will there be 20 grams of the substance left? Leave your answer in terms of \log .

Good Luck!!