

The City College of New York / STEM Institute

THE CITY COLLEGE DEPARTMENT OF MATHEMATICS

SPRING 2010

MATH 20100

FINAL EXAMINATION

Instructor: Dario Cardenas

TA: Isaiah Yim

INSTRUCTIONS:

1. Turn-off cell phones and put them and all notes out of sight.
2. NO CALCULATORS OR ELECTRONIC DEVICES may be used during the exam.
3. NO scrap paper (use sheets provided).
4. Answer each question in the space provided. If you need extra space please use the back of each sheet.
5. Leave all numbers in exact form (Simplify answers when reasonable, but leave them in terms of  $\pi$ ,  $\sqrt{\quad}$ , and fractions.
6. You must show work justifying all answers.

PART 1: [pages 2 to 9] Answer ALL questions in this part. (70 points)

1) (16 points) Find  $\frac{dy}{dx}$ . Simplify your answers in a) and b).

a)  $y = \frac{1 + \sqrt{x}}{1 - \sqrt{x}}$

b)  $y = (x^2 - 1)(x^2 + 2)^2$

c)  $y = \sin(xy) + x^2y$

d)  $y = \sec(\cos x^5)$

2) (16 points) Compute each of the following integrals:

a)  $\int \frac{(1+2x)(1-2x)}{\sqrt{x}} dx$

b)  $\int \frac{3(x+1)}{[3+(x+1)^2]^2} dx$

$$c) \int \frac{-4x}{\sqrt{1-3x^2}} dx$$

$$d) \int_{\pi^2/36}^{\pi^2/16} \frac{(\sec \sqrt{x})(\tan \sqrt{x})}{\sqrt{x}} dx$$

3) (10 points)

a) Using the definition of the derivative as a limit, compute  $f'(x)$  if

$$f(x) = \frac{x}{1-3x}. \text{ (No credit will be given for any other method).}$$

b) Find an equation of the tangent line to the graph of  $f(x) = \frac{x}{1-3x}$  at  $x = 0$ .

- 4) (8 points) A ball is thrown upward from an initial height of 96 feet above the ground with an initial velocity of 16 feet per second. Find:
- the maximum height of the ball.

b) the velocity with which the ball hits the ground.

5) (10 points) Evaluate the following limits or explain why the limit does not exist:

a)  $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x^2 + x - 2}$

b)  $\lim_{x \rightarrow 0} \frac{\sin x [1 - \cos^2(x^2)]}{x^4}$



6) (10 points) For the function  $f(x) = \frac{x^2 - 4}{x^2 - 1}$ , we have  $f'(x) = \frac{6x}{(x^2 - 1)^2}$  and

$$f''(x) = \frac{-6(1 + 3x^2)}{(x^2 - 1)^3}. \text{ (You do not have to verify this).}$$

Using the given information, sketch the graph of  $f(x)$ . Include in your graph the following elements, clearly labeled and justified by appropriate calculations: intercepts, horizontal and vertical asymptotes, local maxima and minima, and inflection points, if any.

PART 2 (pages 10 to 14) (30 points) SOLVE 3 COMPLETE PROBLEMS OUT OF 5. If you solve more than three problems please indicate which problems should be omitted from grading.

7) (10 points)

a) Using differentials or linear approximations, estimate the value of  $\sqrt[3]{32.5}$ . (You may leave your answer in decimal form and/or fractional form).

b) Using a Riemann sum with four equal subintervals and right endpoints, estimate the value of  $\int_0^4 (x^2 + x + 1) dx$ .

- 8) (10 points) Water runs into a right conical tank at the rate of  $36 \text{ cm}^3/\text{min}$ . The tank stands point down and has a height of 8 cm and a base diameter of 16 cm. How fast is the water level rising when the water is 3 cm deep?

9) (10 points) What is the lightest (minimum surface area) open-top right circular cylindrical can that will hold a volume of  $125 \text{ cm}^3$ ?

10)(10 points)

a) Find the value or values of  $c$  that satisfies the conclusion of the Mean

Value Theorem for the function  $f(x) = x + \frac{1}{x}$  on the interval  $\left[\frac{1}{3}, 3\right]$ .

b) Given the function  $f(x) = \frac{1}{\sqrt{x}}$ , find a point  $c$  in  $[1, 4]$  that satisfies the Mean Value Theorem for Definite Integrals.

11) (10 points) Let  $F(x) = \int_{2x}^{3x} \sqrt{t + \cos t} dt$ .

a) What is the value of  $F'(0)$ ?

b) What is the value of  $F''(0)$ ?